Transmission Resonance in Mid-Infrared Left-Handed Metamaterials

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I. Introduction and Background

Objectives

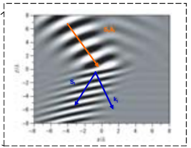
1. To find a correlation between transmission resonance
(a) width and incident angle and
(b) height and incident angle

2. Create a reference for evaluating resonance broadening in novel left-handed metamaterials (LHMs)

Negative Refraction

- Occurs in strongly anisotropic left handed metamaterials for transverse magnetic (TM) polarized light, in which the Poynting vector, S, propagates at a large angle to the wavevector, k

Left-handed metamaterial (LHM): Alternating layers of highly doped InGaAs and intrinsic AlInAs on InP substrate

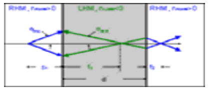


Numerical calculations demonstrating negative refraction of a monochromatic TM polarized Gaussian beam across an air-metamaterial interface

Image and caption: Hoffman et. al. *Nature Mat. Lett.* **6**, 946-950 (2007).

- Result of simultaneous negative permittivity, ε, and permeability, μ. The refractive index, N, can be expressed by the equation $N = \pm\sqrt{\varepsilon\mu}$

- Potential applications include planar birefringent lenses and subwavelength imaging



Adapted from Eleftheriades et. al. *IEEE Trans. on Microwave Theory and Techniques*. **50**, 2702-2712 (2002).

Sketch of planar birefringent lens, able to produce high resolution images

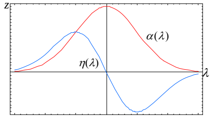
Transmission Resonance

The wavelengths for which negative refraction occurs exhibit a resonance in transmission, due to strong absorption

As derived from the Kramers-Kronig relation:

$$\boxed{\eta(\lambda) \propto \frac{d}{d\lambda}\alpha(\lambda)}$$

$\eta(\lambda)$ is the refractive index as a function of wavelength
$\alpha(\lambda)$ is the absorption as a function of wavelength

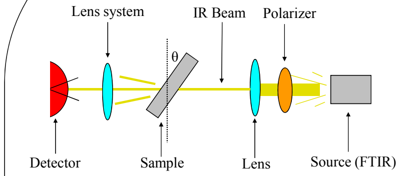


Graph of absorption resonance and corresponding refractive index; note fluctuations of the refractive index in this region

Therefore, a broader transmission resonance will result in weaker fluctuations of the refractive index

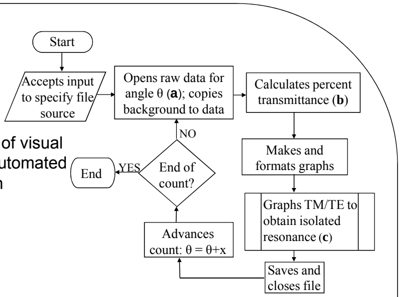
II. Methods

Experimental Setup



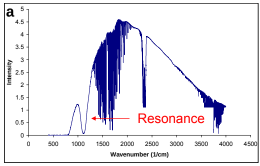
Fourier Transform Infrared Spectrometer (FTIR) configuration

(Right) Flowchart of visual basic macro for automated data interpretation

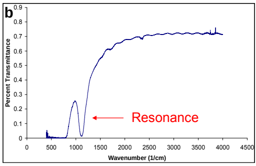


Data Interpretation

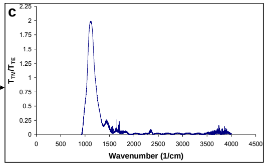
Sample: 20µm thick alternating highly doped InGaAs and intrinsic InAlAs on InP substrate



Transmittance of sample at 50° incidence with TM polarized light



Percent transmittance of sample at 50° incidence with TM polarized light



T_{TM}/T_{TE} of sample at 50° incidence; isolated transmission resonance

Analysis

Full Width at Half Maximum (FWHM) Derivation

Gaussian Equation:

$$f(x) = ae^{-\left(\frac{(x-b)}{c}\right)^2} + d$$

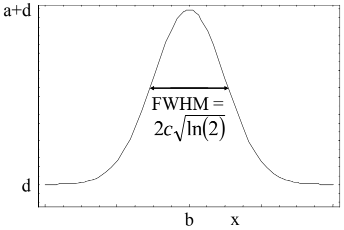
Half Maximum:

$$\frac{f(b)}{2} = \frac{ae^{-\left(\frac{(b-b)}{c}\right)^2}}{2} = \frac{a}{2}$$

FWHM:

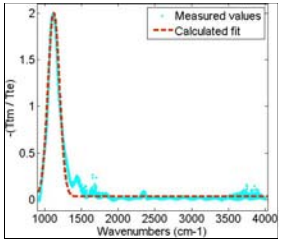
$$ae^{-\left(\frac{(x-b)}{c}\right)^2} = \frac{a}{2}$$

$$2|x-b| = \boxed{2c\sqrt{\ln(2)}}$$



Gaussian curve depicting constants from equation:

- **a**: maximum value of data
- **b**: x value where a occurs
- **c**: determined by MATLAB curve fitting tool; dependent on other parameters
- **d**: obtained from baseline correction formula



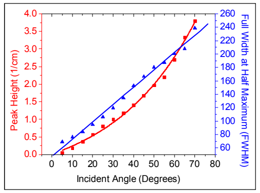
Gaussian fit, $f(x) = 1.97e^{-((x-1125)/108.1)^2} + .03$ with $r^2 = .98$, for measured values of transmission resonance in sample at 50° incidence; FWHM = 180 cm^{-1}

Baseline Correction Derivation:

$$d = \lim_{x\to\infty} f(x) = \frac{1}{n}\sum_{i=x}^{n} f(i) = \boxed{\frac{1}{5970}\sum_{i=1500}^{7469} f(i)}$$

III. Results

Analyzing Resonance Characteristics



Peak Height versus Incident Angle

Equation of exponential growth fit:

$$\hat{f}(x) = .83e^{\frac{x}{40.90}} - .81$$

$$r^2 = .99$$

FWHM versus Incident Angle

Equation of regression line:

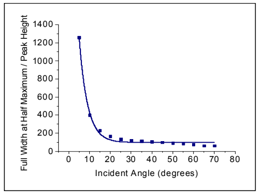
$$\hat{f}(x) = 47.12 + 7.59x$$

$$r^2 = .99$$

Minimal fluctuations of refractive index occur when:

Peak height is low
FWHM is large

∴ Large FWHM / Peak Height ratio is ideal



FWHM / Peak Height versus Incident Angle

Equation of exponential decay fit:

$$\hat{f}(x) = 3990.49e^{-\frac{x}{19.98}} - 15.90$$

$$r^2 = .99$$

IV. Conclusions

- ➢ There is a positive linear correlation between FWHM and incident angle.
- ➢ There is a positive superlinear correlation between peak height and incident angle.
- ➢ **The proposed reference for evaluating new LHMs is the FWHM / Peak Height ratio. A high ratio is ideal for resonance minimization.**
- ➢ *Further Study:* New LHMs engineered to maximize resonance width will be evaluated against existing designs

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